CAUCHY-RIEMANN GEOMETRY AND SUBELLIPTIC THEORY

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CR structures are a bundle theoretic recast of the tangential Cauchy-Riemann equations $\overline{\partial}_b u = 0$ and may be thought of as a geometric framework for their study. Any strictly pseudoconvex CR manifold M endowed with a contact form θ carries a natural second order subelliptic (of order 1/2) operator, the *sublaplacian* Δ_b of (M, θ). Consequently the equations $\Delta_b \phi^i + \sum_{a=1}^{2n} \left(\Gamma^i_{jk} \circ \phi\right) X_a(\phi^j) X_a(\phi^k) = 0$ describing all S^1-invariant harmonic maps $\Phi = \phi \circ \pi : C(M) \to N$ from the total space $C(M)$ of the canonical circle bundle $S^1 \to C(M) \xrightarrow{\pi} M$ endowed with the Fefferman metric F_θ (a Lorentz metric on $C(M)$) form a nonlinear subelliptic system of variational origin. The present talk is devoted to the description of the impact of subelliptic theory on the study of geometric objects appearing on a strictly pseudoconvex CR manifold, such as *subelliptic harmonic maps* and morphisms. For instance let $S^\nu = \{x = (x_1, \cdots, x_{\nu+1}) \in \mathbb{R}^{\nu+1} : x_1^2 + \cdots + x_{\nu+1}^2 = 1\}$ and let $\Sigma \subset S^\nu$ be a codimension 2 totally geodesic submanifold. A continuous map $\phi : M \to S^\nu$ *meets* Σ if $\phi(M) \cap \Sigma \neq \emptyset$. Let $\phi : M \to S^\nu$ be a map that doesn't meet Σ. Then ϕ *links* Σ if the map $\phi : M \to S^\nu \setminus \Sigma$ is null-homotopic. Then a nonconstant subelliptic harmonic map $\phi : M \to S^\nu$ of a compact strictly pseudoconvex CR manifold M into a sphere S^ν either links or meets Σ.

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